Triangle / Sawtooth VCO with voltage controlled continously variable symmetry

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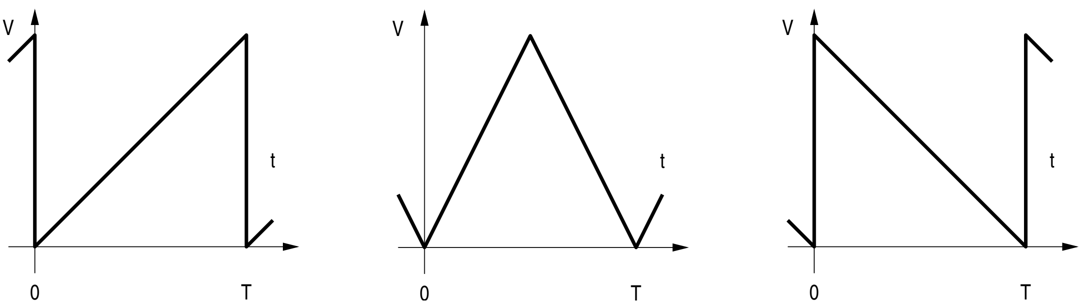
Draft Copy!

A discussion of how to get continously variable symmetry (ratio of the up and the down part to the period of the waveform) from a sawtooth or triangle VCO.

1 Objective

The circuit should be able to produce a waveform that is continously variable via a control voltage (CV) from sawtooth through triangle through to inverted sawtooth while maintaining the frequency set independently by another CV. Without loss of generality we assume that the CV V_W controls



Saw wave (up):

$$V_W = V_{W,\text{min}}$$
$$w = 0$$
$$v = -1$$

Triangle wave:

$$V_W = \frac{1}{2} V_{W,\text{max}}$$
$$w = \frac{1}{2}$$
$$v = 0$$

Inverted saw wave (down):

$$V_W = V_{W,\text{max}}$$
$$w = 1$$
$$v = +1$$

Figure 1: Waveforms at different values of the control voltage and modulation variables

the ratio of the downward slope time versus the period so that at zero control voltage the waveform becomes a sawtooth, a triangle wave is produced at $V_W = \frac{1}{2} V_{W,\text{max}}$ and an inverted sawtooth is produced at maximum V_W. Instead of dealing directly with the CV we introduce a modulation variable w that is normalized to the maximum CV.

$$w = \frac{V_W}{V_{W,\mathrm{max}}} \qquad | \quad V_W \geq 0 \tag{1}$$

$$T = t_u + t_d = (1-w)T + wT \qquad | \quad w \in (0,1) \tag{2}$$

Alternatively we may consider a triangle wave at zero modulation and modulate towards a saw or inverted saw with positive and negative modulation, respectively. The modulation variable is bipolar in this case.

$$v = \frac{2V_W - (V_{W,\mathrm{max}} + V_{W,\mathrm{min}})}{V_{W,\mathrm{max}} - V_{W,\mathrm{min}}} \tag{3}$$

$$T = t_u + t_d = \frac{1+v}{2}T + \frac{1-v}{2}T \qquad | \quad v \in (-1,1) \tag{4}$$

2 Dual Slope Integrator VCO

A sawtooth integrator VCO charges a capacitor with constant current I_{Saw} until an upper threshold voltage $\hat{U}$ is reached. The capacitor is then discharged as quickly as possible to the lower voltage $\check{U}$. For a triangle VCO twice the current is used into the integrator capacitance for the same oscillation frequency and upon reaching the upper threshold the direction of current is reversed to discharge the capacitance to the lower threshold. We are going to use a dual slope integrator with different currents for the charge and discharge phase.

$$T = \frac{\hat{U} - \check{U}}{CI_{\mathrm{Saw}}} \tag{5}$$

$$t_u = \frac{\hat{U} - \check{U}}{CI_u} \tag{6}$$

$$t_d = \frac{\hat{U} - \check{U}}{CI_d} \tag{7}$$

$$T\, I_{\mathrm{Saw}} = t_u I_u \qquad = t_d I_d \tag{8}$$

in particular

$$2I_{\mathrm{Saw}} = I_{\mathrm{Triangle}} \tag{9}$$

and with (2), (3) and (9)

$$I_{\mathrm{Saw}} = (1-w)I_u \; = wI_d \tag{10}$$

$$I_{\mathrm{Triangle}} = (1+v)I_u \; = (1-v)I_d \tag{11}$$

which leads to

$$I_u = \frac{I_{\mathrm{Saw}}}{1-w} \qquad = \frac{I_{\mathrm{Triangle}}}{1+v} \tag{12}$$

$$I_d = \frac{I_{\mathrm{Saw}}}{w} \qquad = \frac{I_{\mathrm{Triangle}}}{1-v} \tag{13}$$

$$\frac{I_u}{I_d} = \frac{w}{1-w} \qquad = \frac{1-v}{1+v} \tag{14}$$

$$1 = \frac{(1-w)I_u}{wI_d} = \frac{(1+v)I_u}{(1-v)I_d} \tag{15}$$

further to

$$I_\Sigma = I_u + I_d \qquad = \frac{I_{\mathrm{Saw}}}{w(1-w)} \tag{16}$$

$$I_\Sigma = I_u + I_d \qquad = \frac{2I_{\mathrm{Triangle}}}{1-v^2} \tag{17}$$

and finally to

$$
\begin{aligned}
I_u &= wI_\Sigma &&= (1-\nu)I_\Sigma && (18)\\
I_d &= (1-w)I_\Sigma &&= (1+\nu)I_\Sigma && (19)
\end{aligned}
$$

We observe that the charge and discharge currents are ratiometric to each other in proportion to the applied modulation, their sum is always larger than four times the charge current of a simple sawtooth generator while the individual currents can never become smaller than the charge current of the sawtooth generator. Errors in the up and down charge currents will change the symmetry and the frequency of the oscillator. It is possible to trade the frequency error in for an amplitude error by means of an alternative control circuit. In both cases the error can be converted to a signal that could be used in a feedback loop to correct it.

It also becomes clear that the modulation range has to be restricted as otherwise the sum current (through one of the individual currents) would become infinite. Independently of how the charge and discharge currents are produced, it proves quite difficult to keep the frequency constant through the modulation. However if additionally triangle and/or sawtooth waves are produced from the same CV it might be more interesting musically to let the phase of the oscillators drift with respect to each other instead of locking them in DCO style.

3 Dual Sawtooth Waveshaper

The waveshaper uses an arrangement of zero based[1] up and down sawtooth of the same frequency and phase. These are amplified by an appropriate amount and the output waveform is produced as the minimum of the two amplified sawtooths. Hence

$$
\begin{aligned}
V_{\mathrm{Saw,u}} &= a_u \frac{t \mod T}{T} && (20)\\
V_{\mathrm{Saw,d}} &= a_d \left(1 - \frac{t \mod T}{T}\right) && (21)
\end{aligned}
$$

are up and down sawtooth waveforms with amplitudes a_u and a_d, respectively. These two quantities can also be regarded as gain factors for sawtooths with their amplitude normalized to one. The output waveform is obtained as

$$
V_{\mathrm{Out}} = \min(V_{\mathrm{Saw,u}}, V_{\mathrm{Saw,d}}) \tag{22}
$$

The switch from the up to the down sawtooth happens when their momentaneous amplitude is equal, which leads to

$$
\begin{aligned}
\frac{t_{ud} \mod T}{T} &= \frac{a_d}{a_u + a_d} && (23)\\
V_{\mathrm{out}}(t_{ud}) &= \frac{a_u a_d}{a_u + a_d} && (24)
\end{aligned}
$$

In order for the output amplitude to stay constant at one it is necessary for the two gain factors to satisfy

$$
1 = \frac{1}{a_u} + \frac{1}{a_d} \tag{25}
$$

Thus the up and down times of the combined waves are

$$
T = \frac{1}{a_u}T + \frac{1}{a_d}T \tag{26}
$$

[1] It is possible to keep them symmetric around zero, but then one needs to offset by $2aV_{\mathrm{peak}}$ when a gain of a is applied.

With (2) and (3) we get

$$a_u = \frac{1}{1-w} = \frac{2}{1+v} \tag{27}$$

$$a_d = \frac{1}{w} \quad = \frac{2}{1-v} \tag{28}$$

which we find to correspond to (12) and (13) normalized to I_{Saw}. This is welcome news as it means the two methods can use almost the same circuitry and all the derivations in 2 can be used by simply normalizing them to I_{Saw}. Note that by construction any errors in one or both of the gains alter both the amplitude and the symmetry of the output wave, but not it's frequency.

4 Implementation

Don Tillmans's brilliant idea[2] saves the multiplier otherwise necessary to satisfy (25). In the context of the waveshaper this keeps the amplitude constant throughout the modulation. Don Tillman observed that the identity

$$1 = \frac{1}{1+e^x} + \frac{1}{1+e^{-x}} \tag{29}$$

holds unconditionally and leads to

$$a_u = 1 + e^x \tag{30}$$

$$a_d = 1 + e^{-x} \tag{31}$$

This has an almost trivial implementation as a bipolar transistor circuit. However, as we have seen above this circuit does not only work in the context of the waveshaper – we could also use it to produce the two currents for a dual slope integrator VCO.

As proposed originally, however, there are no provisions for linear response of the symmetry modulation, more specifically the modulation gets slower at the end of the modulation range. To make the modulation linear with the CV we use (27) and (28) to arrive at

$$x = \ln\frac{w}{1-w} = \ln\frac{1-v}{1+v} \tag{32}$$

which again has a straight-forward circuit implementation. Putting the two together we find that we're implementing a special configuration of a Log-Antilog-Multiplier. We cannot use an analog multiplier IC directly, however, as for instance the RC4200 would only implement (15), which is underdetermined as we can fix only two of the four variables. This means we'd need to use yet another multiplier to determine the third variable, for instance to calculate (12).

Instead, we can use a discrete log-ratio amplifier to obtain

$$x_d = -V_T \ln\frac{1+v}{1-v} \tag{33}$$

$$x_u = -V_T \ln\frac{1-v}{1+v} \tag{34}$$

simultaneously by using just one additional opamp, then proceed to de-logarithmize this

$$y_d = U_{ref}\exp\left(\frac{-x_d}{V_T}\right) = U_{ref}\frac{1+v}{1-v} \tag{35}$$

$$y_u = U_{ref}\exp\left(\frac{-x_u}{V_T}\right) = U_{ref}\frac{1-v}{1+v} \tag{36}$$

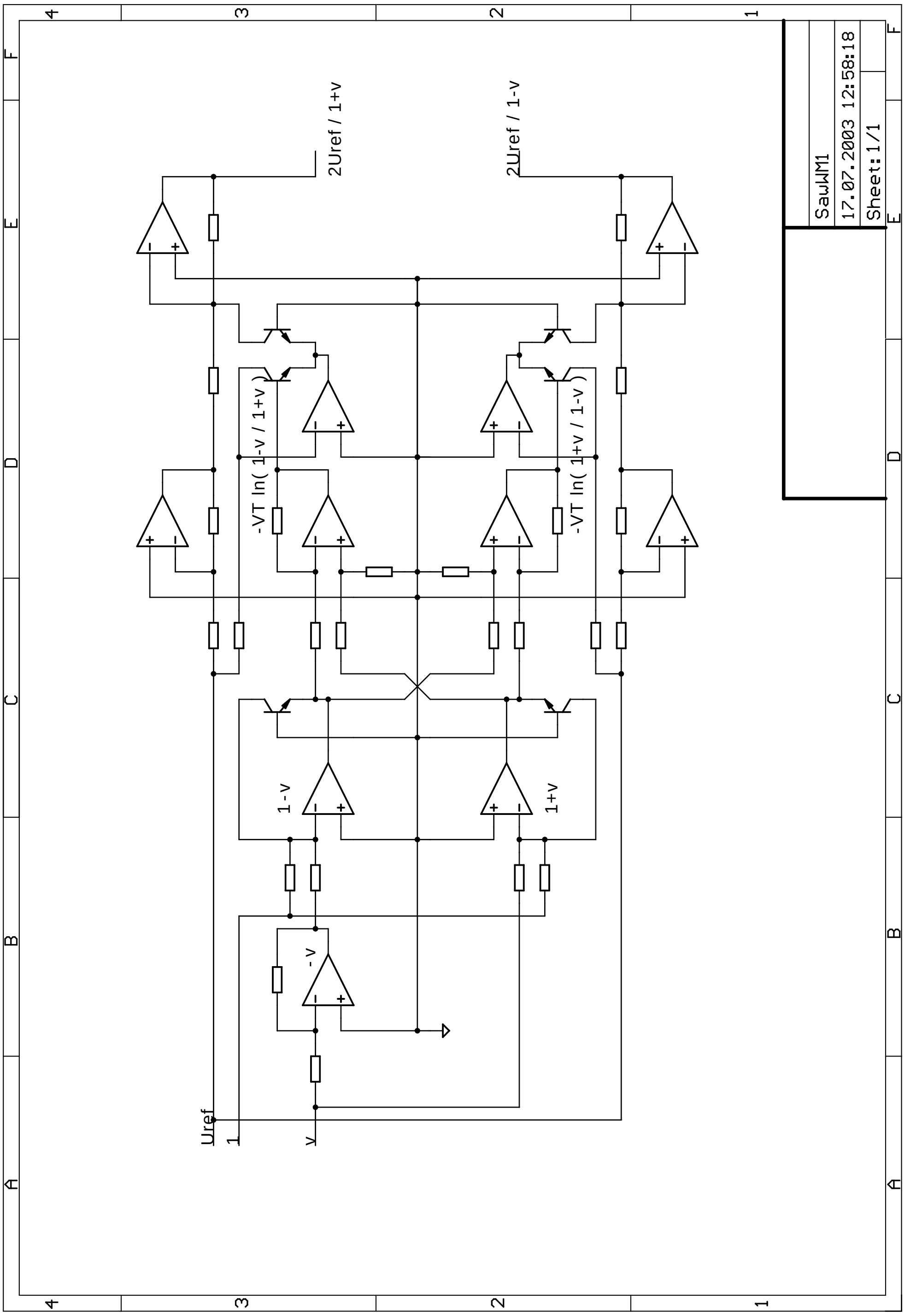
[2]http://www.till.com/articles/VariableSaw/index.html

and finally add one in the form of $\frac{U_{\mathrm{ref}}}{U_{\mathrm{ref}}}$ to obtain

$$U_d = \frac{2U_{\mathrm{ref}}}{1-\nu} \tag{37}$$

$$U_u = \frac{2U_{\mathrm{ref}}}{1+\nu} \tag{38}$$

which compares favorably with (12), (13) and (27), (28). A rough sketch showing a circuit along those lines is appended. None of the necessary compensation elements for the opamps are shown. However there is no overall temperature dependance to first order if all transistors are kept at equal temperature. Note that by replacing the difference amps of the log-ratio amplifier by two two-quadrant multipliers with their gain input tied together, the response curve of the symmetry modulation can be continously varied from sub- to superlinear.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Sheet: 1/1
2Uref / 1+v
2Uref / 1-v
-VT ln(1-v / 1+v)
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